Preliminaries

For a digital image \( f(x,y) \) the basic filtering equation is

\[
g(x, y) = \mathcal{F}^{-1}\{ H(u, v)F(u, v) \}
\]

Where \( H(u,v) \) and \( F(u,v) \) are DFTs of the image and of the filter. Their product is defined by array (element-by-element) multiplication. Specifications of \( H(u,v) \) are simplified considerably when using functions symmetric about their centers. This requires that \( F(u,v) \) is also centered, which is accomplished by multiplying the input image by \((-1)^{x+y}\) before computing its transform.
Simple filters

Considering the following image with centered (and scaled) DFT

One of the simplest filters would be $H(u,v)$ having 0 at the center and 1 elsewhere. Such filter will reject the DC (constant) term and leave everything else unchanged.

Since the DC term represents an average intensity, setting it to zero results in reduction of average intensity to zero. Therefore, the resulting image appears darker. Actually, aero average implies negative intensities.
Simple filters

Low frequencies in the transform are related to slowly varying intensity components of image. High frequencies are caused by sharp transitions in intensity (edges, noise).

Therefore, LPF will blur an image reducing details (and noise).

HPF will enhance sharp details but cause reduction in contrast since it eliminates DC component. Adding a small constant to a HPF does not affect sharpening properties but prevents elimination of DC term and, thus, preserves contrast.

Simple filters

\[
\begin{align*}
H(u, v) & \quad \text{Normal} \\
\alpha = 0.85 & \quad \text{Example}
\end{align*}
\]
Zero-padding

Periodicity implied by DFT leads to a "wraparound" error: data from adjacent periods aliases leading to incorrect results of convolution. To avoid "wraparounds", convolved functions must be zero-padded.

More on zero-padding

Blurred images are not uniform: top white edges are blurred but side edges are not (without zero-padding). The zero-padding leads to an expected result.
More on zero-padding

Due to periodicity implicit while using DFT, a spatial filter passing through the top edge of the image encompasses a part of the image and also a part of the bottom of the periodic image right above it. Padding helps to avoid this...

Zero-padding and ringing

Considering an ideal LPF and its spatial representation obtained via multiplication by (-1)^n and IDFT, we notice that zero-padding will lead to discontinuities. If we compute DFT of zero-padded filter (to obtain its frequency characteristic), ringing (Gibbs?) will occur. Ideal – infinite sinc – truncation – ringing!
Appearance of ringing

Results of filtering with ILPF… Notice ringing appearance.

We cannot use ideal filters AND zero-padding at the same time! One approach would be to zero-pad images and then create filters (of the same zero-padded size) in frequency domain. However, this will produce (insignificant though) wraparounds since the filter won’t be zero-padded.

On phase angle

Since DFT of an image is a complex array, say

\[ F(u, v) = R(u, v) + jI(u, v) \]

The filtered image would be

\[ g(x, y) = \mathcal{F}^{-1} \{ H(u, v)R(u, v) + jH(u, v)I(u, v) \} \]

We are interested in filters affecting the real and imaginary parts equally – zero-phase-shift filters. Even small changes in the phase angle may have drastic effect!
Summary of steps

1. For the input image \( f(x,y) \) of size \( M \times N \), form the zero-padded image \( f_p(x,y) \) of size \( P \times Q \) (typically \( P = 2M \), \( Q = 2N \)), where
   \[
   P \geq 2M - 1; \quad Q \geq 2N - 1
   \]
2. Obtain \( f_c(x,y) \) multiplying \( f_p(x,y) \) by \((-1)^{x+y}\) to center its transform;
3. Compute the DFT of the \( f_c(x,y) \);
4. Generate a real, symmetric filter function \( H(u,v) \) of size \( P \times Q \) with center at coordinates \((P/2, Q/2)\);
5. Form the product \( G(u,v) = H(u,v) F(u,v) \) via array multiplication;
6. Obtain the processed image
   \[
   g_p(x,y) = \left( \text{real}\left[ \mathcal{F}^{-1}\{G(u,v)\} \right] \right)(-1)^{x+y}
   \]
7. Finally, extract \( g(x,y) \) – the \( M \times N \) region from the top left quadrant of \( g_p(x,y) \).
Spatial vs. Frequency domains

The link between filtering in spatial and in frequency domains is the convolution theorem...

\[ h(x, y) \Leftrightarrow H(u, v) \]

However, DFT implements a circular convolution.

Considering next the image and its spectrum (magnitude of its DFT) of size 600 x 600

We attempt to use a 3x3 Sobel mask… Therefore, we need to zero-pad both the image and the mask to size of 602 x 602 (keeping the mask at the center to preserve symmetry) to avoid wraparounds. The filter In frequency domain… Results of frequency-domain filtering and spatial filtering are same.